

DERIVATION OF PRACTICAL RELIABILITY-BASED POST-FIRE ASSESSMENT TOOLS FOR STRUCTURAL ELEMENTS



Ruben Van Coile¹



Robby Caspee²



Luc Taerwe³

ABSTRACT

A practical reliability-based post-fire assessment method has been presented in earlier work which evaluates the maximum allowable characteristic value of the imposed load effect on concrete elements after being subjected to fire. The method is known as the ReAssess method and can be extended to cover the post-fire assessment of other material types, structural members and limit states. This extension however requires knowledge of the methodology underlying the derivation of the basic equations. In this paper a detailed overview of this methodology is presented for the first time, incorporating all the steps from the definition of the limit state problem up to the verification of the reliability obtained when applying the method. The presented methodology allows for the development of practical reliability-based post-fire assessment tools to a broad range of structural members. An example derivation is given illustrating the application of the presented concepts to the post-fire bending limit state for a simply supported concrete slab.

1 INTRODUCTION

Most fires are not so severe as to result in failure of the structural elements and generally the structure will still be standing after the fire has been extinguished or has burned out. When large residual deformations and damage to the connections are obvious the structure may have to be demolished, but in many other situations the post-fire visual inspection may indicate the possibility of continued post-fire use. In those situations the maximum load which can be safely carried by the structural elements after fire has to be determined. A practical reliability-based method has been proposed in [1]. This reliability-based post-fire assessment method (ReAssess method) uses analytical formulas and a pre-calculated diagram to determine the post-fire maximum allowable characteristic value of the imposed load which corresponds with a specified target safety level. Applications of the method to concrete elements have been presented e.g. in [1] and [2], but no detailed overview has yet been presented of the methodology which underlies the derivation of the

¹ Postdoctoral Research Associate. BRE Centre for Fire Safety Engineering, School of Engineering, University of Edinburgh. UK. Magnel Laboratory of Concrete Research, Department of Structural Engineering, Ghent University, BELGIUM. e-mail: ruben.vancoile@ed.ac.uk. **Corresponding Author.**

² Professor. Magnel Laboratory of Concrete Research, Department of Structural Engineering, Ghent University. BELGIUM.

³ Professor. Magnel Laboratory of Concrete Research, Department of Structural Engineering, Ghent University. BELGIUM

analytical formulas for application in the ReAssess method. A clear overview of the methodology and associated assumptions allows for the application of the ReAssess method to other material types, elements, and load situations. Allowing for this generalization is the goal of this paper. The paper starts by presenting the ReAssess method in Section 2 with an example application on the post-fire bending assessment of a concrete slab. Section 3 presents a flowchart with the underlying methodology for deriving the material and limit state dependent equations for application of the ReAssess method, and Section 4 shows how the flowchart has been applied to derive the equations used for the example application of Section 2. Conclusions are given in Section 5.

2 EXAMPLE CASE

Consider a fully developed fire occurring in one of the flats of an apartment building. After the structure has cooled down small residual deflections of the ceiling and cracks can be observed. As the ceiling functions as the floorplate of the apartment above, an assessment must be made to evaluate whether it is safe for the occupants of the apartment above to continue using their property. This evaluation is done in accordance with the flowchart given in *Fig. 1*. Its application will be illustrated further.

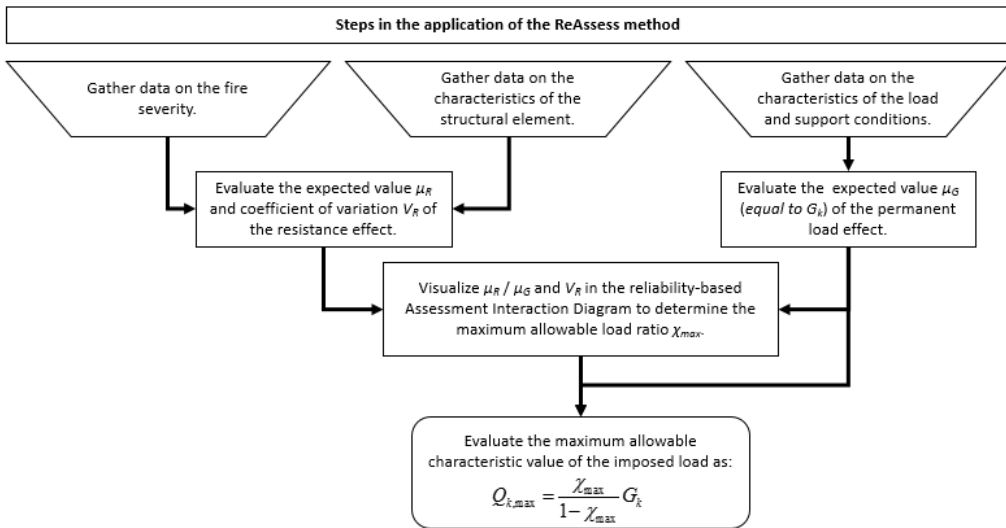


Fig. 1. Flowchart indicating the different conceptual steps in the application of the ReAssess method

As indicated in *Fig. 1*, the first steps consist of gathering data on the fire severity, slab characteristics, load and support conditions. Based on an inspection, the slab is simply supported and the uniformly distributed permanent load associated with the self-weight of the slab and finishes equals $g_k = 6 \text{ kN/m}^2$. The fire severity is assessed as an equivalent ISO 834 standard fire duration t_E in function of the fire load density, compartment dimensions and ventilation characteristics through the equivalency rule given in Annex F of EN 1991-1-2 [3]. The fire load density for an apartment is however uncertain and in absence of additional information, the nominal fire load density and associated uncertainty (Gumbel distribution) specified in Annex E of EN 1991-1-2 for dwellings is used. The resultant equivalent fire durations and associated probabilities p_{IE} are given in *Table 1*. Improved assessments for the fire severity can be made through expert judgement, modelling or measurements, see e.g. [2]. The characteristics of the slab are evaluated

considering the as-built drawings and considering standard uncertainties given in literature [4]. This data is given in *Table 2*. If required, an improved assessment can be made of the data in *Table 2* by performing measurements (for example concrete compressive strength assessment by the destructive testing of cores taken from an undamaged section of the slab). However, by applying the ReAssess method before executing tests, the utility of these tests can be evaluated prior to their commissioning. If required, the ReAssess method can be applied iteratively as new information becomes available.

Table 1. Equivalent ISO 834 fire duration and associated probability

t_E [min]	15	30	45	60	75	90
p_E [-]	0.01	0.39	0.42	0.14	0.03	0.01

Table 2. Slab characteristics and standard uncertainties based on [4]

Parameter	Symbol	Dimension	Distribution	Mean μ	Standard deviation σ
20°C concrete compressive strength ($f_{ck} = 30\text{MPa}$)	$f_{c,20}$	MPa	Lognormal	42.9	6.4
20°C reinforcement yield stress ($f_{yk} = 500\text{MPa}$)	$f_{y,20}$	MPa	Lognormal	581.4	40.7
reduction factor for the residual reinforcement yield stress after heating to temperature θ	$k_{fy,res}$	-	Beta [$\mu-3\sigma$; $\mu+3\sigma$]	1.00*	0.05*
slab thickness	h	mm	Normal	200	5
bottom reinforcement area	A_s	mm ²	Normal	785.4	4.5
bottom reinforcement axis distance to exposed surface	a	mm	Beta [$\mu-3\sigma$; $\mu+3\sigma$]	40	5
slab width (unit width)	b	mm	Deterministic	1000	-
slab span	l	m	Deterministic	6	-
permanent load ($g_k = 6\text{kN/m}^2$)	g	kN/m ²	Normal	6	0.6
imposed load ($q_{k,max}$ to be determined)	q	kN/m ²	Gumbel	$0.6q_k$	$0.21q_k$
total model uncertainty	K_T	-	Lognormal	1.11	0.16
model uncertainty for the resistance effect	K_R	-	Lognormal	1.1	0.1
model uncertainty for the load effect	K_E	-	Lognormal	1.0	0.1

* temperature dependent, see discussion below.

The residual yield stress reduction factor $k_{fy,res}$ is dependent on the maximum temperature attained by the reinforcement bars. This maximum temperature both depends on the fire considered (i.e. the ISO 834 standard fire duration) and the stochastic reinforcement position. With a thermal calculation tool [1] the temperature ingress in the concrete slab is evaluated for each of the t_E of *Table 1* and the associated reinforcement temperatures are determined considering a discretization of the range of possible axis positions in 5 equally wide zones with respective probabilities p_{ai} . For each zone median axis position a_i and fire duration t_E , the reinforcement temperature is determined and the associated expected value and standard deviation of $k_{fy,res}$ are evaluated in accordance with the residual material model listed in [1], based on [5]. These t_E and a_i dependent values for $\mu_{kfy,res}$ and $\sigma_{kfy,res}$ are combined considering the probabilities p_{tE} and p_{ai} , using *Eq. (1)* and *(2)*. Further discussion is given in [1].

$$\mu_{kfy,res} = \sum_{t_E, a_i} \mu_{kfy,res}(t_E, a_i) p_{tE} p_{ai} \quad (1)$$

$$\sigma_{kfy,res} = \sqrt{\sum_{t_E, a_i} \left(\left(\mu_{kfy,res}(t_E, a_i) - \mu_{kfy,res} \right)^2 p_{tE} p_{ai} \right) + \sum_{t_E, a_i} \sigma_{kfy,res}^2(t_E, a_i) p_{tE} p_{ai}} \quad (2)$$

Considering the flowchart in *Fig. 1*, the next step in application of the ReAssess method is the evaluation of the expected value μ_R and coefficient of variation V_R of the ‘combined resistance effect’ R , as well as the expected value μ_G of the load effect induced by the permanent loads. For the permanent load effect, the expected value is equal to the characteristic value [4], resulting in $\mu_G = g_k \cdot l^2/8 = 27\text{kNm}$ for the mid-span bending moment. The parameters μ_R and V_R are assessed through *Eq. (3)–(6)*, where (3) is the analytical model for the combined resistance effect (including the total model uncertainty K_T), and (4)–(6) are Taylor approximations. Furthermore, μ is the vector of mean values for the respective stochastic variables X_i of *Table 2*. When incorporating the methodology in standards or guidance documents the analytical Taylor approximations *Eq. (4)–(6)* would be written in full, but here this has been omitted for conciseness.

$$R = K_T A_s k_{fy, res} f_{y, 20} \left(h - a - \frac{A_s k_{fy, res} f_{y, 20}}{2 f_{c, 20} b} \right) \quad (3)$$

$$\mu_R \approx R(\mu) = 78.4\text{kNm} \quad (4)$$

$$\sigma_R \approx \sqrt{\sum_i \left(\frac{\partial R(\mu)}{\partial X_i} \right)^2 \sigma_{X_i}^2} = 13.3\text{kNm} \quad (5)$$

$$V_R = \frac{\sigma_R}{\mu_R} = 0.17 \quad (6)$$

Having evaluated the equations above, the point defined by $\mu_R / \mu_G = 2.90$ and $V_R = 0.17$ is positioned in the Assessment Interaction Diagram (AID) of *Fig. 2*, applicable for a target reliability index $\beta_{t, 50} = 3.8$ considering a 50 year reference period t_{ref} . Interpolation in the AID gives $\chi_{max} = 0.34$, with χ the ‘load ratio’ defined by $q_k / (q_k + g_k)$. The target reliability index $\beta_{t, 50} = 3.8$ corresponds with the target of EN 1990 for design for new structures, assuming that no reduction in safety is accepted when assessing the post-fire usability as compared to the design of new structures. Applying the last step in the flowchart of *Fig. 1*, the maximum allowable characteristic value $q_{k, max}$ of the imposed load is evaluated through *Eq. (7)* as 3.09kN/m^2 . This maximum allowable characteristic value far exceeds the required 2kN/m^2 for dwellings and thus this first assessment concludes that the flat above the fire apartment can be further occupied while investigations and repair works progress.

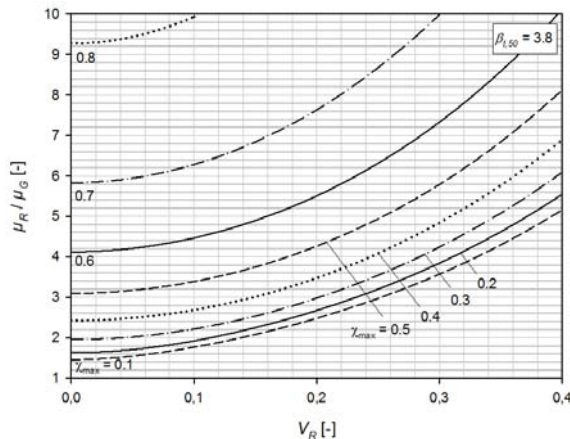


Fig. 2. Assessment Interaction Diagram (AID) for $\beta_{t, ref} = 3.8$ and $t_{ref} = 50$ years

$$q_{k,\max} = \frac{\chi_{\max}}{1 - \chi_{\max}} g_k = \frac{0.34}{1 - 0.34} 6 \frac{kN}{m^2} = 3.09 \frac{kN}{m^2} \quad (7)$$

3 METHODOLOGY FOR MATERIAL AND LIMIT STATE SPECIFIC EQUATION

The example above illustrates the application of the ReAssess method for the post-fire assessment of a simply supported concrete slab. This application is however dependent on the availability of the analytical formula (3), or on elaborated versions of Eq. (4)-(6) in a standard or guidance document, and furthermore require knowledge on the total model uncertainty K_T . These have been evaluated for a limited number of cases in [1]. However, by following the different steps in the flowchart of Fig. 3, the ReAssess method can be applied to other limit states (e.g. shear) and materials (e.g. concrete filled hollow section steel columns).

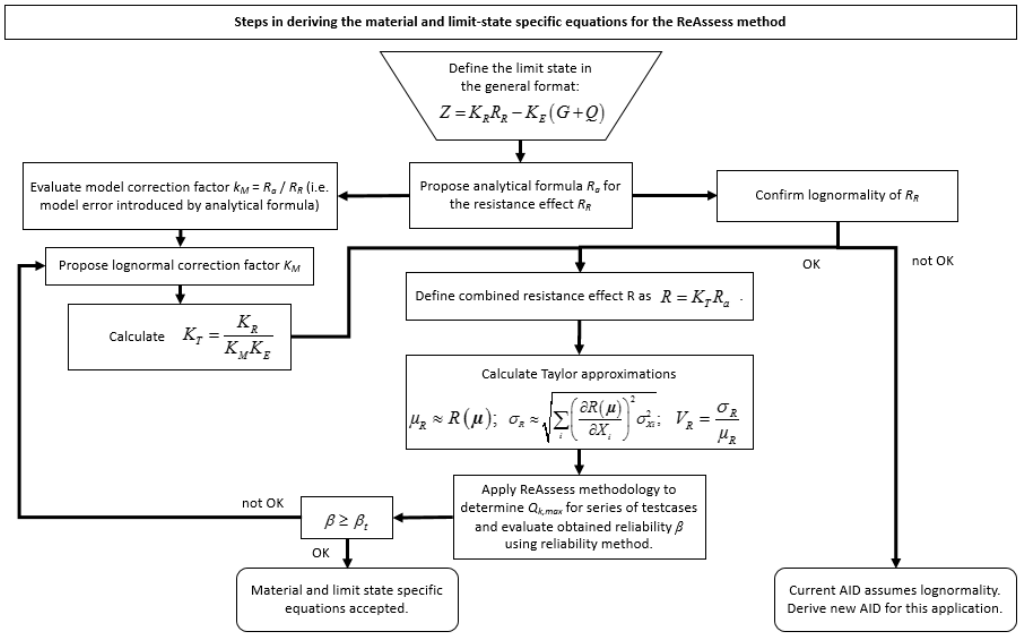


Fig. 3. Flowchart for deriving material and limit-state specific equations for application in the ReAssess method

The first step consists of a definition of the limit state in the general format as indicated in Fig. 3, with K_R and K_E the lognormal model uncertainties for the resistance and load effect as listed in [4] for different limit states, R_R the resistance effect (i.e. the member capacity), G the permanent load effect and Q the imposed load effect. When the limit state cannot be written in this format, the current AID cannot be applied and further developments are required. The second step requires the proposal of an analytical formula describing the resistance effect. As this analytical formula generally results in a deviation from the ‘true’ resistance effect (as evaluated by a numerical model or tests), the model correction factor k_M is evaluated and a lognormal correction factor K_M is proposed which is conservative across different situations. As the AID in Fig. 2 has been derived assuming lognormality for the resistance effect R_R , this assumption must be verified as well to ensure the applicability of the AID. If the lognormality assumption is not fulfilled, in principle different Assessment Interaction Diagrams have to be derived or other modifications have to be

made. Having defined a lognormal correction factor K_M , the total model uncertainty K_T is analytically determined and the combined resistance effect R is given by $K_T R_a$. Further applying the original ReAssess methodology μ_R and V_R are evaluated using Taylor approximations, resulting in easy-to-use analytical formulas. To validate the obtained equations and K_T , the maximum allowable imposed load should be determined for a number of test cases and the corresponding reliability indices β should be evaluated using reliability methods. These obtained reliability indices should not be lower than the target reliability index β_t corresponding with the applied AID (i.e. $\beta_t = 3.8$ for a 50 year reference period for the AID of Fig. 2). If this requirement is not fulfilled, the proposed model for K_M can be adjusted to make a more conservative estimate of the combined resistance effect. When the requirement is fulfilled, the derived equations and total model uncertainty K_T can be applied together with the AID for performing a reliability-based post-fire assessment.

4 APPLICATION TO THE EXAMPLE CASE

4.1 Step 1: Definition of the limit state

Considering a simply supported slab subjected to pure bending, the applicable limit state is given by Eq. (8), with M_R the post-fire residual bending moment capacity, M_G the bending moment induced by the permanent load effect, and M_Q the bending moment induced by the imposed load effect.

$$Z = K_R M_R - K_E (M_G + M_Q) \quad (8)$$

4.2 Step 2: Proposal of an analytical formula for the resistance effect

An analytical approximation $M_{R,a}$ for the residual bending moment capacity is given by Eq. (9), derived from Fig. 4. The model in Fig. 4 is based on the limiting isotherm concept as incorporated in EN 1992-1-2 [6] for the structural fire design of concrete elements. In Fig. 4 the same concept is applied as part of a post-fire assessment. As illustrated by Fig. 4, the limiting isotherm concept neglects the strength contribution of concrete heated to temperatures above the limiting isotherm θ_{lim} while the compressive strength of less heated concrete is modelled by the initial 20°C compressive strength $f_{c,20}$. The reinforcement strength contribution is incorporated considering the locally attained temperature of the reinforcement.

$$M_{R,a} = A_s k_{fy,res} f_{y,20} \left(h - a - \frac{A_s k_{fy,res} f_{y,20}}{2 f_{c,20} b} \right) \quad (9)$$

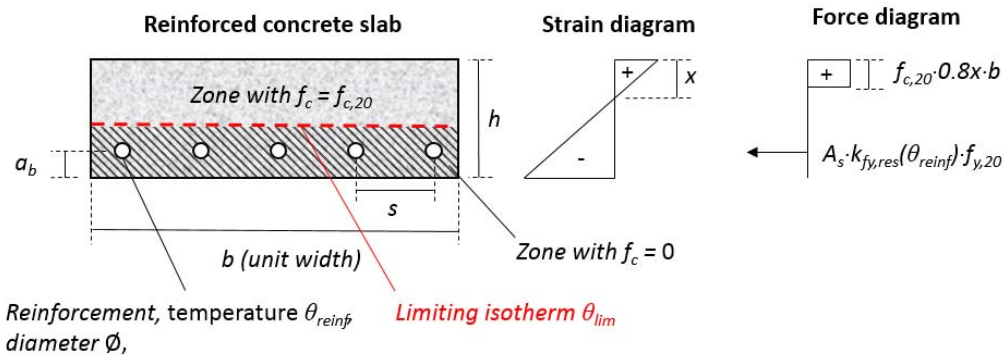


Fig. 4. Concept cross-section limiting isotherm, and strain and force diagram for the residual bending moment capacity

4.3 Step 3: Confirm lognormal distribution for M_R

Monte Carlo simulations are performed considering the parameter models given in *Table 2*, using the detailed numerical model presented in [1] which takes into account the local residual mechanical properties of both the concrete and the reinforcement. Results are visualized in *Fig. 5a*. Statistical tests indicate that these histograms can be approximated by a lognormal distribution for ISO 834 standard fire durations up to 150min. For the considered slab configuration a 150min fire is very severe and the probability of failure during fire significantly increases for more severe fires, making the application of the ReAssess method for more severe fires less likely. Furthermore, note that the investigated slab configuration has a 120min structural fire resistance in accordance with the tabulated data in EN 1992-1-2 [6] when applied as a one-way load-bearing slab.

4.4 Step 4 and 5: Model correction factor k_M and proposal for the lognormal variable K_M

The model correction factor k_M is evaluated by considering coupled Monte Carlo simulations. On the one hand the residual bending capacity M_R is evaluated using the detailed numerical model (as described in [1]). On the other hand the same parameter realizations are implemented in the analytical approximation $M_{R,a}$ of *Eq. (9)*. The ratio of both gives a histogram for k_M as visualized for different ISO 834 durations t_E in *Fig. 5b*. This graph also visualizes the proposed lognormal model correction factor K_M ($\mu_{KM} = 1.004$ and $V_{KM} = 0.003$) which is considered conservative across different t_E . Note that the model correction factor is very close to unity, indicating the excellent approximation by the analytical model for the considered limit state.

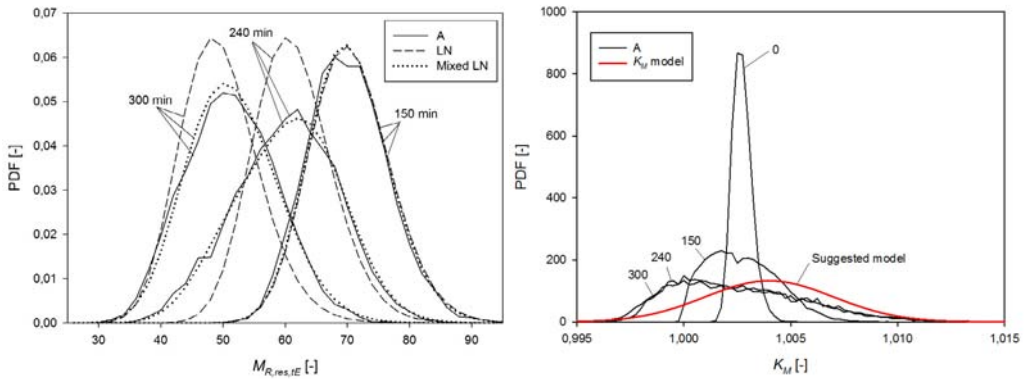


Fig. 5. (a) Observed histogram of $M_{R,a}$ for different ISO 834 durations t_E ; (b) Observed histogram for the model correction factor k_M and proposed lognormal model K_M

4.5 Step 6: Calculate the total model uncertainty K_T

The total model uncertainty K_T is evaluated as $K_T = K_R / (K_E \cdot K_M)$. As both the model uncertainties and the correction factor are defined by a lognormal distribution (see *Table 2* for K_R and K_E), K_T is described by a lognormal distribution as well and its parameters are defined analytically. The obtained result has been given earlier in *Table 2*.

4.6 Step 7 and 8: Define $R = K_T R_a$ and evaluate μ_R and V_R using Taylor approximations

As indicated in *Fig. 3*. Guidance documents may prefer to list the fully elaborated equations.

4.7 Step 9: Apply the ReAssess method for series of test cases and evaluate $\beta \geq \beta_t$

The derived equations are intended to result in a reliability index β at least as high as the target reliability index β_t specified in the Assessment Interaction Diagram (AID). Evaluating μ_R and V_R for the slab configuration of *Table 2*, the AID is applied for a wide range of hypothetical values for $\mu_G = M_{Gk}$, resulting in corresponding maximum allowable values for $M_{Qk,max}$. By applying the First Order Reliability Method (FORM), the reliability index β is evaluated which is obtained when applying these combinations of M_{Gk} and $M_{Qk,max}$ together with the residual capacity M_R in the limit

state of $Eq. (8)$, for different fire durations t_E . Results are visualized in *Fig. 6*, confirming that the application of the proposed equations and total model uncertainty K_T within the ReAssess method indeed results in a reliability index β exceeding the target value of 3.8. *Fig. 6* also visualizes the reliability index β obtained for new slabs designed in accordance with the Eurocode guidance. As indicated, the ReAssess method is found to result in a more precise approximation of the target β .

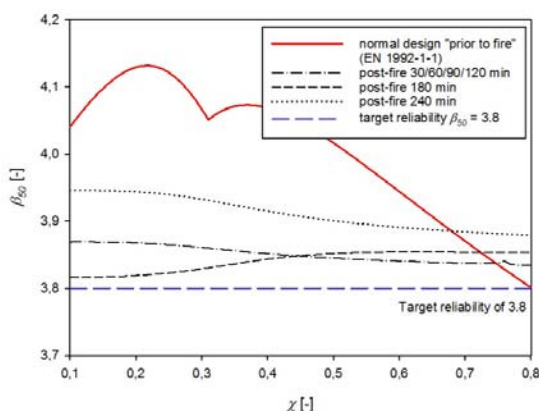


Fig. 6. Reliability index β obtained with the ReAssess method for different load ratios $\chi = M_{Qk} / (M_{Gk} + M_{Qk})$ and for different fire durations t_E

5 CONCLUSIONS

A practical reliability-based assessment method (ReAssess method) has been proposed in earlier contributions for the post-fire assessment of concrete elements. In order to extend the application to other material types and limit states, the methodology for deriving the material and limit state specific equations has been presented together with a flowchart indicating the tests which have to be performed to ensure the applicability of the newly derived equations within the existing ReAssess method. The presented methodology gives direct guidance on the development of reliability-based post-fire assessment tools for all types of structural elements, and is in principle independent of the material type and limit state considered. Applying the methodology presented in this paper will allow to develop post-fire assessment tools which make a fast and reliable evaluation of the post-fire maximum allowable characteristic value of the imposed load on a structural element.

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